EAI ADHOCNETS 2014 Conference
Keynote Presentation

Applications of MRFs in Distributed Complex Networks

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Outline

- Overview of (distributed) complex networks
- Recurring problems in distributed complex communications networks
- Markov Random Fields (MRFs)
  - Objectives and approach
- Applying MRFs in distributed complex networks
  - Network formation
  - Malware propagation
  - Power control
  - Resource allocation & cross-layer design
- Directions for future work
- Discussion

Complex Communications Networks

- Set of interacting entities
  - **Collaborating** $\rightarrow$ coalitions, or **Competing**
- Emerging trade-off:

gain vs. **cost** of collaboration or interaction

- Complex Networks (CNs)
  - Wide range of systems of interacting entities (actors, nodes, etc.)
  - Each node performs some complex computation (simple or sophisticated)
  - Diverse topologies, but same features of interest
  - Model similar problems in different settings

Network Science and Communications Networks

- Emerging {common + generic} problems in CNs
- A complex network theory required
- Mathematical models for diverse networks and their emerging problems

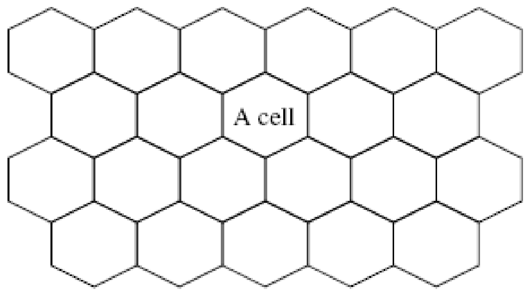
  Study of similar {statistical, social, structural} properties & behaviors

- Working examples:
  - Spreading of a disease in a social network
  - Malware diffusion over a telecommunication network
  - Information dissemination in an affiliation network
  - Failure propagation in a large power network
  - Financial crisis spreading in global markets

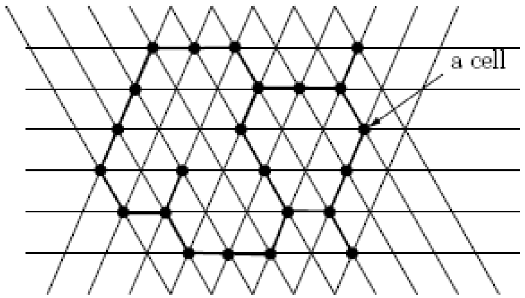
They all describe the same fundamental problem ⇒ Network Science

Types of Complex Networks of Interest

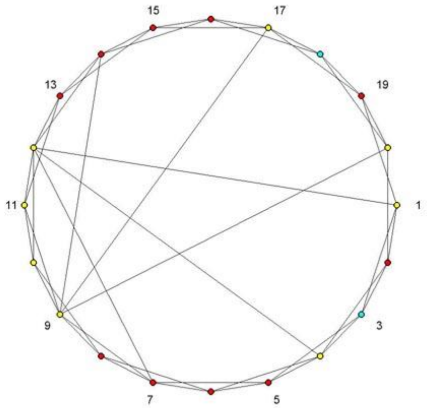
- **Regular**
  - Circular
  - Grids
  - Mesh
- **Random** (Generalized Random)
  - Erdos-Renyi model variations
- **Small-world** (Watts-Strogatz model)
- **Scale-free** (Barabasi-Albert model)
  - Exponential degree distribution
- **Random geometric** (multi-hop)
  - Ad hoc, sensor, vehicular, etc.

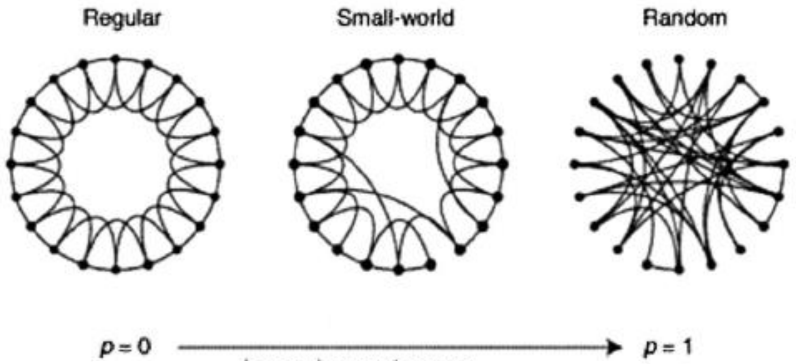


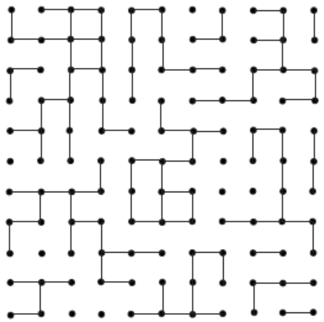
(a) cellular network

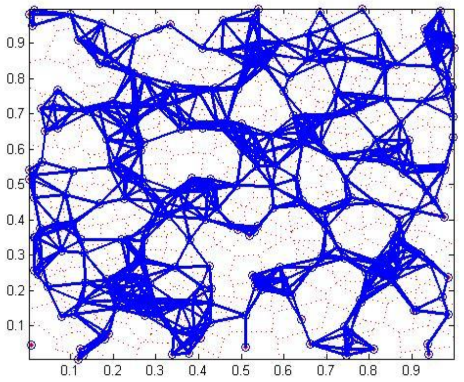


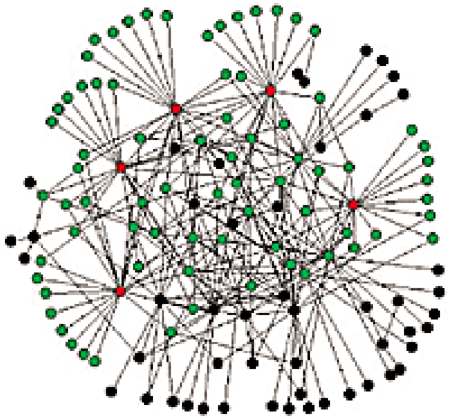
(b) triangle-free cellular network











Summary of Social Metrics of Interest

Network type	Degree distribution	Average Path Length	Centrality
Regular	dirac function	constant	constant
S-W	heavy-tailed	small	varying
S-F	power-law	small	varying
RG	Poisson	average	uniform
RGG	uniform	large	uniform

Fundamental Recurring Problems in Communications

- Emerging & recurring problems emerge in all Network Science disciplines
- Focus on those emerging in communications networks:
  - Network formation & network growth
  - Distributed computation
  - Reliability & robustness
  - Resource allocation & cross-layer design
  - Scaling – stability
  - Network management
  - Implementation complexity and cost of operation
- Addressed via diverse analytical/simulation methodologies

Traditionally Employed Methodologies

- Optimization
  - Convex programming
  - Integer programming
  - Quadratic optimization
  - Nonlinear optimization
  - Multi-objective optimization
- Calculus of variations – optimal control problems
- Probabilistic and combinatorial techniques
- Game theory
- Stochastic Geometry
- Graph theory
- And many others and their variations……

Statistical Mechanics and Communications Networks

- Various mathematical techniques adopted
  - Thermodynamic parameters mapped to heterogeneous packet transmissions
  - Diffusion processes
  - Polymer physics
  - Percolation
  - Brown motion – random walks
- Markov Random Fields (MRFs)
  - Spin glasses
  - Magnetic fields and spins
- MRFs have been used extensively in image processing
  - (since late 1970)

MRFs – Objectives and Approach

- Avoid global optimization via local decision-making
  - Exchange info with local neighbors
  - Progressively propagate info of local neighbors to the whole network
- Successive convergence to optimizers while avoiding local traps
  - E.g. stochastic optimization – simulated annealing
- As close as possible to global optimizers – achieve them if possible
- Sequential and parallel implementation
- Convergence (?) to global optimizers is the main issue

Random Fields and MRFs

- Random Field (RF): A collection (set) of **random variables** $\{X_i\}$
  - Each r.v. describes the state of an entity (**site**, node, etc.) $X_i = x_i$
  - State can be binary or multi-valued → **phase space** Λ_s and **state space** Λ
- **Configuration** over a **neighborhood** system $\omega = \{(x_1, \ldots, x_s, \ldots, x_n) : x_s \in \Lambda, s \in S\}$
  - One of possible states of the system
- RF is a strictly positive prob. measure on the state space

$$\mathbb{P}(X_s = x_s \mid X_r = x_r,\ r \neq s)$$

- Markov Random Fields (MRFs) → RFs with **spatial Markov property**

$$\mathbb{P}(X_s = x_s \mid X_r = x_r, r \neq s) = \mathbb{P}(X_s = x_s \mid X_r = x_r, r \in \mathcal{G}_s)$$

- G_s describes the neighborhood system
- Local characteristics depend only on knowledge of state of neighboring sites

MRFs and Gibbs Fields

- Gibbs Field: special form of RF
- Characterized by the **energy function** *U(ω)*
- **Gibbs distribution**:

$$\Pr\left(X=\omega\right)=\frac{1}{Z}e^{-\frac{U(\omega)}{T}}$$

- **Partition function** *Z*:

$$Z \triangleq \sum_{\omega\in\Omega} e^{-\frac{U(\omega)}{T}}$$

  - *T* is the temperature parameter of the system → specifies min. selection sensitivity
- *U(ω)* metric of system ‘energy’ → objective function to minimize
- **Hammersley-Clifford theorem**: Gibbs RF (distribution) with energy function expressed in terms of neighbor potentials is equivalent to MRF and vice-versa

MRFs and Potential Functions

- *U(ω)* decomposed into a family of functions $\rightarrow$ potentials

$$U(\omega) = \sum_{c \in \in C} V_c(\omega)$$

- Typically employed nearest pairwise neighbor potentials
  - Singleton
  - Doubleton
  - 3-cliques, etc.
- Mostly interested in singleton – doubleton potentials
- Applications: Gibbs measures where 3-clique and higher order clique potentials are zero

Stochastic Relaxation: Sequential & Parallel Samplers

- As # of sites increases, state space increases exponentially
  - Direct sampling with Gibbs is intractable $\rightarrow$ partition function
  - Probabilistic space reduction is feasible (e.g. Monte Carlo simulation) $\rightarrow$ stoch. relaxation
- Generate a Markov chain on the configuration with GF as equilibrium distrib.
- log annealing schedule (Simulated Annealing) $\rightarrow$ Gibbs sampling results in configurations with globally min. energy
- Local energy changes permitted $\rightarrow$ avoid traps in local minima
- Implementations
  - Sequential sampler
  - Parallel sampler

MRFs :: Applications

- MRFs applied already in various fields
  - Statistical mechanics
  - Image processing and video analysis
    - Exploit locality of pixel values to determine pixel values

- Applications in communications networks → focus in the following
  - Network formation
  - Malware propagation
  - Power control
  - Resource allocation & cross-layer design

MRFs & Network Formation

- **General problem**: self-organization of distributed/autonomous networks
  - UAV control – commercial and military drones
  - Vehicles in highways
  - Obstacle avoidance and personal assistance
- **Critical challenges:**
  - Achieve global objectives – target locations over the mobility terrains
  - Constrained optimization – obstacles and other mobility requirements
  - Distributed coordination among groups of nodes
  - Wireless signaling among nodes
  - Fast response and adaptation $\Rightarrow$ swarming/flocking inspired by birds, animals, etc.
- **Specific problem:** model swarming via MRFs
  - Previous approaches (artificial potential functions) suffer local min. entrapment

MRF Swarming Formulation

- Discrete possible locations for nodes-sites: $1 \leq i \leq N_x \quad 1 \leq j \leq N_y$
  - States: location coordinates $x_k = (i_k, j_k)$
  - Neighborhood defined via mobility-sensing radius $R_m < R_s$
- Evolution via seq. Gibbs sampling: pick annealing scheme and # of sweeps
  - Determine set of next candidate locations:

$$L_k \triangleq \Lambda_k \cap \{(i,j) : \sqrt{(i-i_k)^2 + (j-j_k)^2} \leq R_m\}$$

  - For each $l \in L_k$ evaluate:

$$\Phi_k(x^l) \triangleq \hat{\Phi}_k(x_k = l, \{x_{k'} : k' \in N_k\}) \qquad P(x_k = l) = \frac{e^{-\frac{\Phi_k(x^l)}{T(n)}}}{\sum_{l' \in L_k} e^{-\frac{\Phi_k(x^{l'})}{T(n)}}}$$

  - Loop back until the # of sweeps
- In parallel implementation, each node computes next move independently

MRF Swarming Scenarios

- Gathering

$$\hat{\Phi}_k(x_k, \{x_{k'} : k' \in \mathcal{N}_k\}) = \lambda_1 \|x_k - z_0\| + \begin{cases} \frac{\lambda_2}{\sum_{k' \in \mathcal{N}_k} \frac{1}{\|x_k - x_{k'}\|}} \text{ if } \mathcal{N}_k \neq \emptyset \\ \Delta \text{ if } \mathcal{N}_k = \emptyset \end{cases}$$

$$T(n) = \frac{1}{4\log(400+n)}$$

- Dispersion

$$\hat{\Phi}_k(x_k, \{x_{k'} : k' \in \mathcal{N}_k\}) = \begin{cases} \frac{\lambda}{\min_{k' \in \mathcal{N}_k} \|x_k - x_{k'}\|} \text{ if } \mathcal{N}_k \neq \emptyset \\ \epsilon \text{ if } \mathcal{N}_k = \emptyset \end{cases}$$

- Line formation

$$\hat{\Phi}_k(x_k, \{x'_k : k' \in \mathcal{N}_k\}) = \begin{cases} \frac{\lambda}{m_k} \sum_{k' \in \mathcal{N}_k} \frac{d_{k,k'}}{R_s} (1 - |\sin(\theta_{k,k'})|)^2 \text{ if } m_k > 0 \\ \Delta \text{ if } m_k = 0 \end{cases}$$

$$T(n) = \frac{1}{4log(400+n)}$$

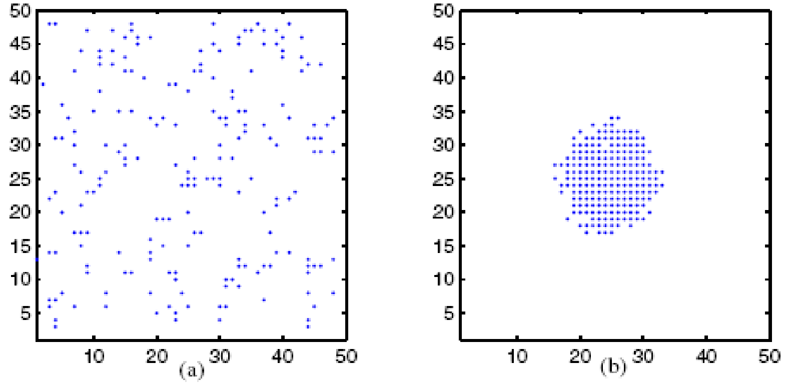
$$\lambda > 0$$

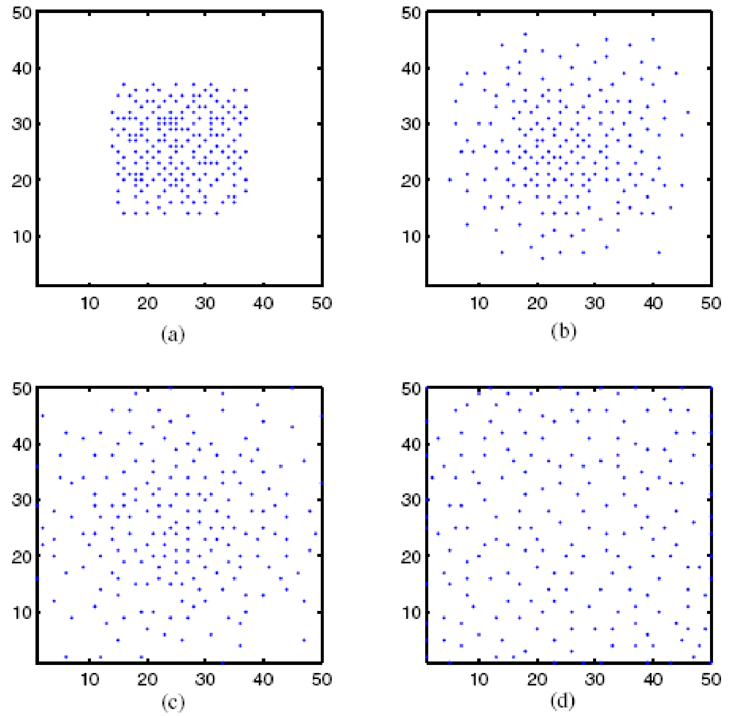
$$\Delta > 0$$

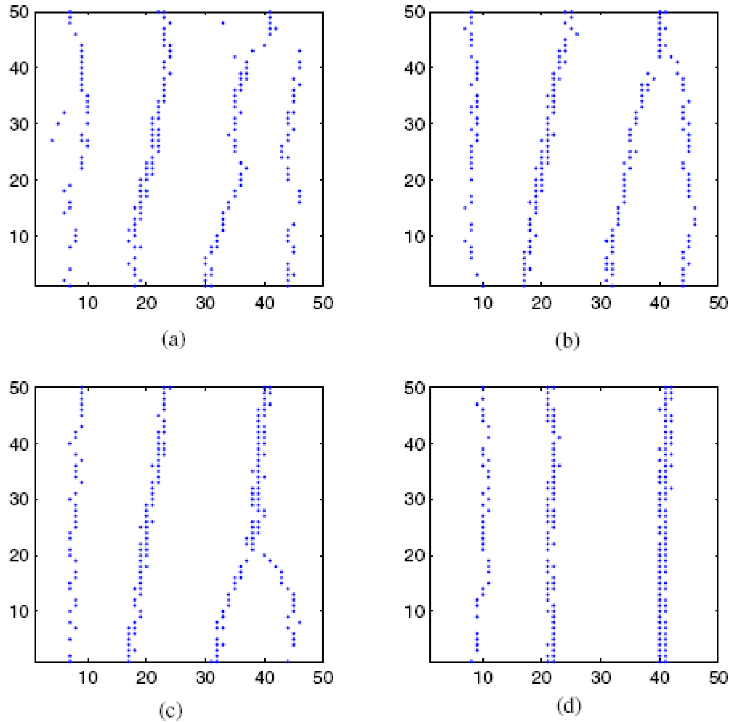
- Weight on farthest neighbors to form longer lines $\frac{d_{k,k'}}{R_s}$

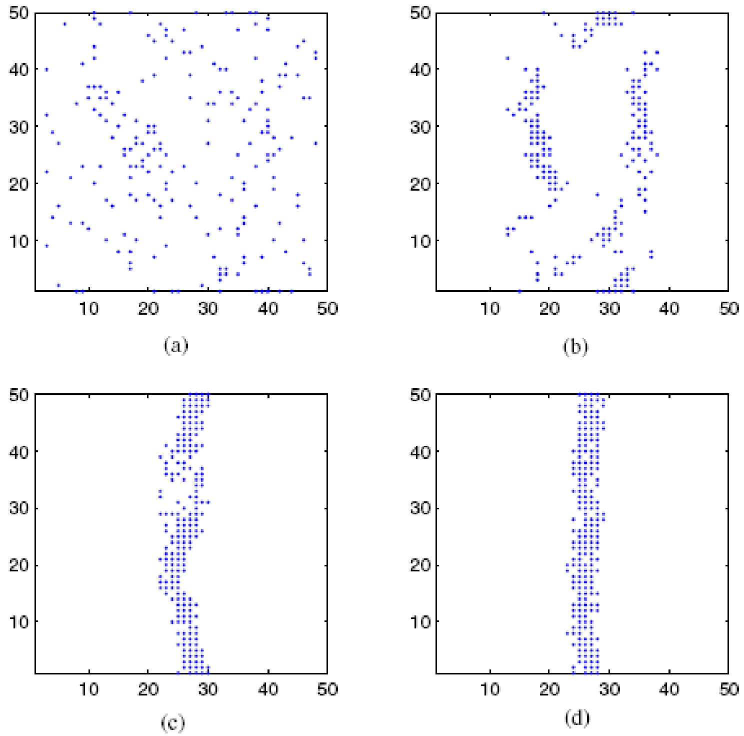
MRF Swarming Indicative Results

- Various examples of swarming via sequential MRF
  - Gathering
  - Dispersion
  - Formation of lines || to *y*-axis









MRFs & Malware Spreading

- **General problem**: model malware propagation in complex networks
  - Classic viruses/worms/etc.
  - Emerging mobile malware
  - Recurrent malware in the long-term (devices prone to receive malware all the time)
- **Critical challenges:**
  - Model properly state transitions
  - Diverse topologies with different features
  - Generic modeling framework
- **Specific problem:** model SIS malware type via MRFs
  - Previous approaches focus on more specific threats than generic malware

MRF Malware Spreading Formulation

- Demonstration for a chain network with neighborhood $\mathcal{N}_k = \{k-1, k+1\}$
  - Binary phase space: infected – susceptible: $\Lambda = \{-1, 1\}$
  - Neighbor potentials and energy function:

$$\Phi_k(x) = \hat{\Phi}_k\left(x_k, \{x_{k'} : k' \in \mathcal{N}_k\}\right) = \hat{\Phi}_k\left(x_k, \{x_{k-1}, x_{k+1}\}\right)$$

$$\Phi_k\left(x_k^1\right) = \hat{\Phi}_k(x_k = 1, x_{k-1}, x_{k+1}) = x_{k-1} + x_{k+1}$$

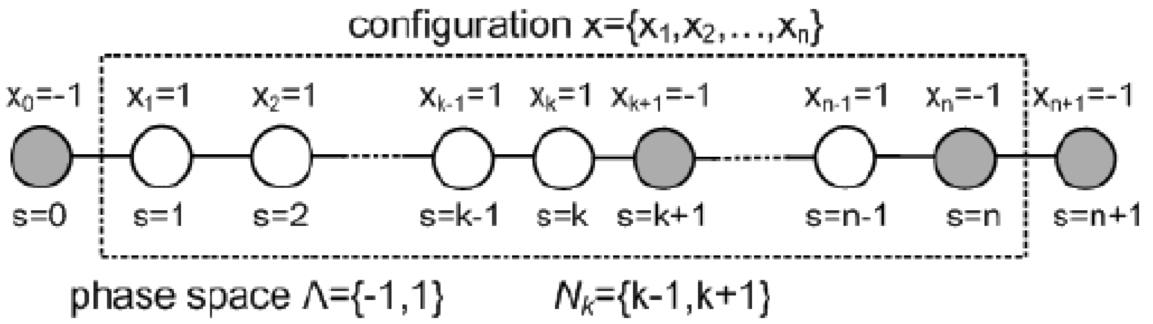
$$U(x) = -J \sum_{k=0}^{n} x_k x_{k+1}$$

- Probabilities for configuration values:

$$\mathbb{P}\left(x_k = \ell\right) = \frac{e^{-\frac{\Phi_k(x_k^{\ell})}{T(n)}}}{\sum_{\ell' \in L_k} e^{-\frac{\Phi_k(x_k^{\ell'})}{T(n)}}}$$

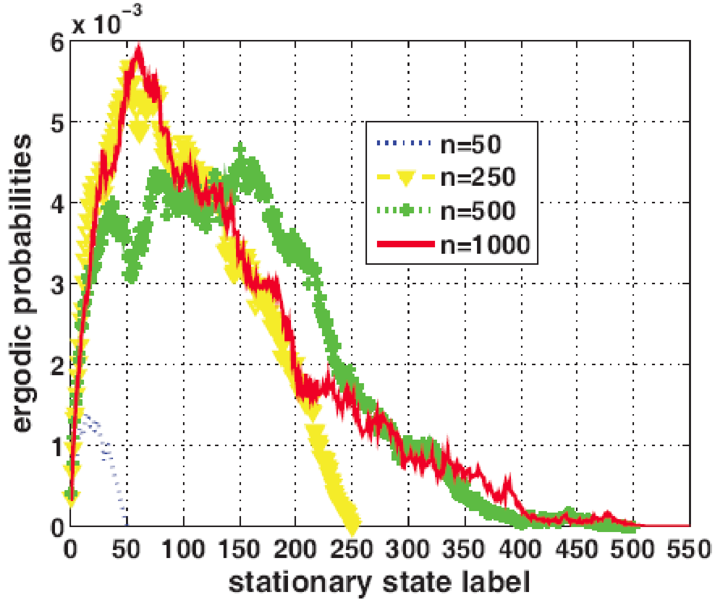
  - Yielding:

$$\mathbb{P}(x_k = 1) = \frac{1}{1 + e^{-\frac{\left(\Phi_k(x_k^{-1}) - \Phi_k(x_k^{1})\right)}{T(n)}}} = \frac{1}{1 + e^{\frac{2(x_{k-1} + x_{k+1})}{T(n)}}}$$

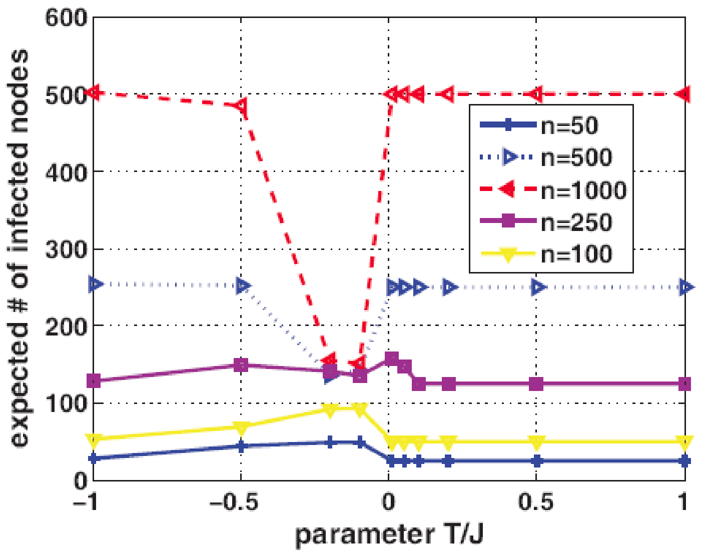


MRF Malware Propagation Solution Features

- 10000 sweeps, 50 averaging scenarios
- Ergodic distributions of system states
  - State: # of infected nodes
  - As the chain increases distributions have longer tails – tougher to infect most nodes



- Expected # of infected nodes
  - Larger networks, more expected infected (not %-wise)
  - Phase transitions emerging w.r.t. *n* and *T/J*
  - For large *n* the drop is more significant
  - For *T/J>0.1* propagation depends mainly on size



MRF Power Control in Wireless Networks

- **General problem**: optimal control of power
  - Control energy consumption
  - Mitigate co-channel interference
  - Maintain connectivity and network robustness
- **Critical challenges for power control:**
  - Non-convex nature: optimal power control very tough via utility maximization
  - Obtain globally optimal solution in a distributed & asynchronous manner
  - Strict assumptions on employed utility functions
- **Specific problem:** distributed optimal control via utility functions
  - Previous approaches (e.g. MAPEL) are centralized – significant computational overhead
  - Mostly converge to suboptimal solutions
  - GLAD $\rightarrow$ optimal solution for general utility functions

Original Power Control Formulation

- MRF sites: set of transmission links $\mathcal{M} = \{1, \cdots, M\}$
  - phase space: set of available power levels $\boldsymbol{p} = (p_i, \forall i \in \mathcal{M})$

$$\boldsymbol{P}^{\min} = (P_i^{\min}, \forall i \in \mathcal{M}) \qquad \boldsymbol{P}^{\max} = (P_i^{\max}, \forall i \in \mathcal{M})$$

  - System utility function as a received SINR function:

$$U(\gamma(\boldsymbol{p})) = \sum_{i=1}^{M} U_i(\gamma_i(\boldsymbol{p})) \qquad \gamma_i(\boldsymbol{p}) = \frac{G_{ii}p_i}{\sum_{j \neq i} G_{ji}p_j + n_i}$$

- Find power allocation max. overall system utility

$$\begin{aligned} \text{UM}: \quad & \underset{\boldsymbol{p}}{\text{maximize}} \quad U(\gamma(\boldsymbol{p})) \\ & \text{subject to} \quad P_i^{\min} \leq p_i \leq P_i^{\max}, \ \forall i \in \mathcal{M}. \end{aligned}$$

  - *U(.)* non-negative and continuous – no other restrictive assumptions

MRF Power Control Formulation

- Gibbs sampling solves the optimization $h^* = \min_{\boldsymbol{x} \in \mathcal{X}} H(\boldsymbol{x})$, $\mathcal{X} = \prod_{n=1}^{N} \mathcal{X}_n \subset \mathcal{R}^N$
- The value of each site updated according to $\Lambda_n(x_n|\boldsymbol{x}_{-n}) = \dfrac{\exp\left(-\beta H(x_n, \boldsymbol{x}_{-n})\right)}{\sum_{x'_n \in \mathcal{X}_n} \exp\left(-\beta H(x'_n, \boldsymbol{x}_{-n})\right)}$

$$\boldsymbol{x}_{-n} = (x_1, \cdots, x_{n-1}, x_{n+1}, \cdots, x_N)$$

- Assuming discrete power levels as the phases of transmission links, the power of each link selected according to:

$$\Lambda_i(p_i|\boldsymbol{p}_{-i}(t_i^{(k)}-)) = \begin{cases} 0, \text{if } U(\gamma(p_i, \boldsymbol{p}_{-i}(t_i^{(k)}-))) = 0 \\ \dfrac{\exp\left(\frac{-\beta}{U(\gamma(p_i, \boldsymbol{p}_{-i}(t_i^{(k)}-)))}\right)}{\sum_{p'_i \in \mathcal{P}_i^D : U(\gamma(p'_i, \boldsymbol{p}_{-i}(t_i^{(k)}-))) \neq 0} \exp\left(\frac{-\beta}{U(\gamma(p'_i, \boldsymbol{p}_{-i}(t_i^{(k)}-)))}\right)}, \text{otherwise} \end{cases}$$

- The SINR is computed by:

$$\gamma_j(p_i, \boldsymbol{p}_{-i}(t_i^{(k)}-)) = \begin{cases} \dfrac{\gamma_j(\boldsymbol{p}(t_i^{(k)}-))p_i}{p_i(t_i^{(k)}-)}, & j = i \\ \dfrac{s_j(t_i^{(k)}-)}{\frac{s_j(t_i^{(k)}-)}{\gamma_j(\boldsymbol{p}(t_i^{(k)}-))} + G_{ij}(p_i - p_i(t_i^{(k)}-))}, & j \neq i, \end{cases}$$

MRF Power Control Solution Features

- Discrete-GLAD
  - Continuous version as well
- Starting from arbitrary initial conf.

D-GLAD → Markov chain

converging to stationary

$$\Omega_\beta(p) = \begin{cases} 0, \textit{if } U(\gamma(p)) = 0; \\ \dfrac{\exp\left(\frac{-\beta}{U(\gamma(p))}\right)}{\sum_{p' \in \mathcal{P}^D : U(\gamma(p')) \neq 0} \exp\left(\frac{-\beta}{U(\gamma(p'))}\right)}, \textit{otherwise}, \end{cases}$$

- The convergence rate is linear in total variational distance

$$||\Omega_\beta^{(k)} - \Omega_\beta||_{var} \leq c_\beta |\lambda_{2\beta}|^k$$

Algorithm 1 The Discrete-GLAD Algorithm

The implementation at each transmitter node T_i

1: **Initialization:** pick a sequence of time epochs $\{t_i^{(1)}, t_i^{(2)}, \cdots\}$ in continuous time.
2: Choose some feasible power $p_i(t_i^{(1)}) \in \mathcal{P}_i^D$. Let $k = 1$.
3: **repeat**
4: Transmit the data packet with the power level $p_i(t_i^{(k)})$.
5: Keep sensing the control packets broadcasted by receivers, and then update the information of γ_j's and s_j's.
6: $k = k + 1$.
7: Update the feasible power $p_i(t_i^{(k)}) \in \mathcal{P}_i^D$ according to the probability distribution given in (5).
8: **until** Link i decides to leave the network

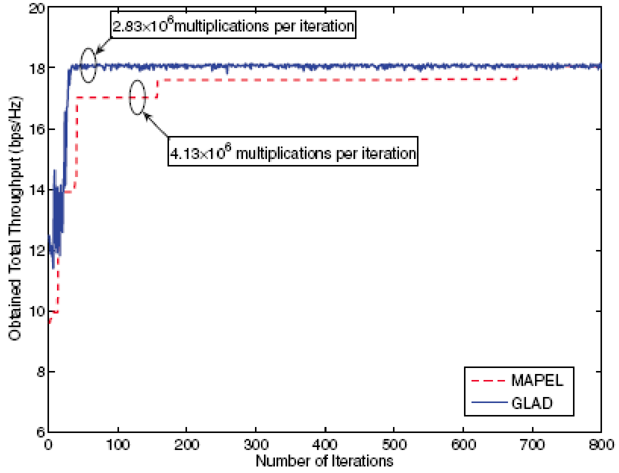
The implementation at each receiver node R_i:

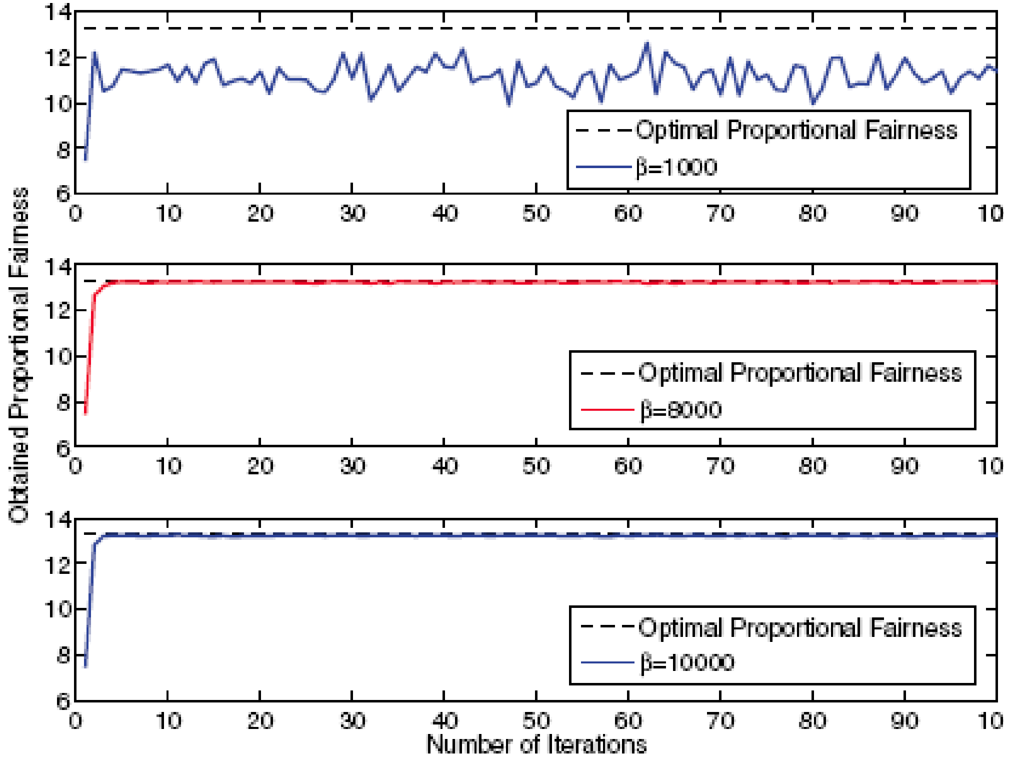
1: **repeat**
2: Keep measuring its received SINR and received power, and broadcast them in a control packet when a change in the SINR or power is sensed.
3: **until** Link i leaves the network

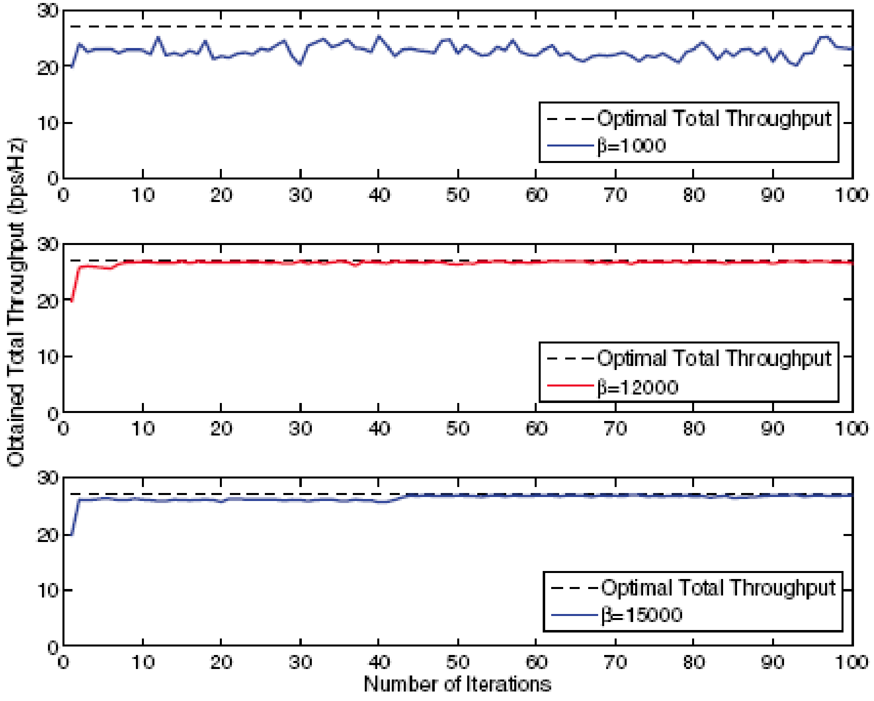
$$\mathcal{P}^D = \{p | p_i \in \mathcal{P}_i^D, \forall i\}$$

MRF Power Control Convergence and Complexity

- Obtained proportional fairness
- Complexity comparison

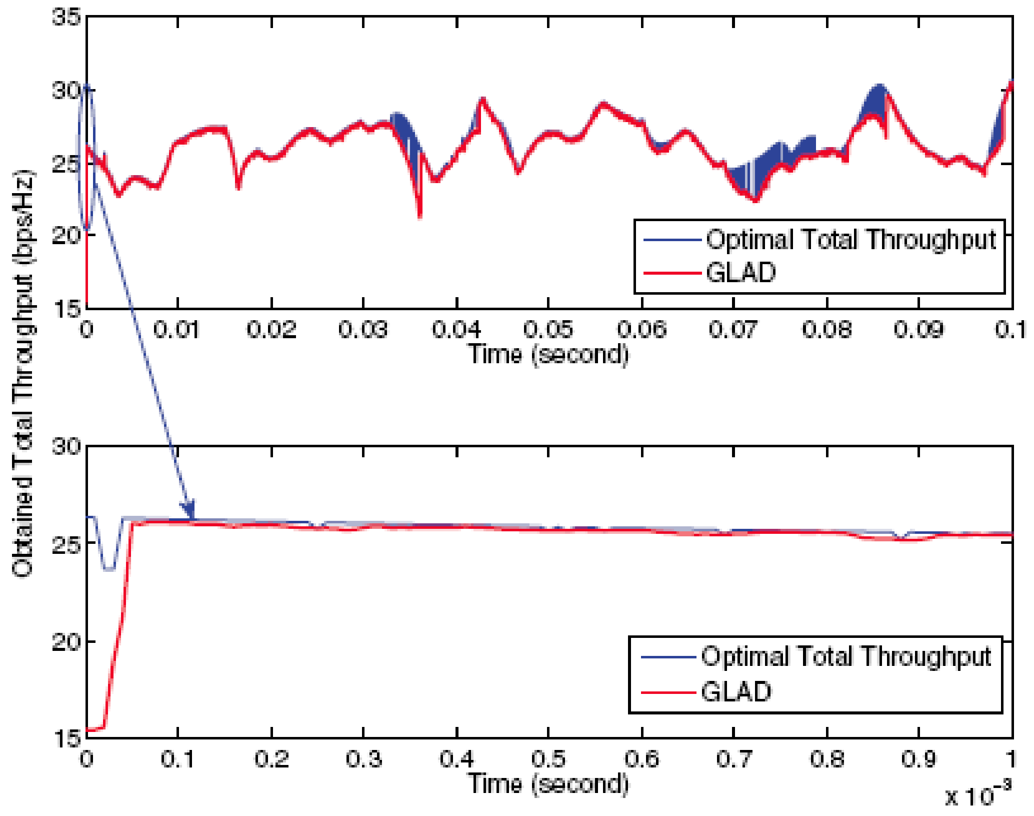


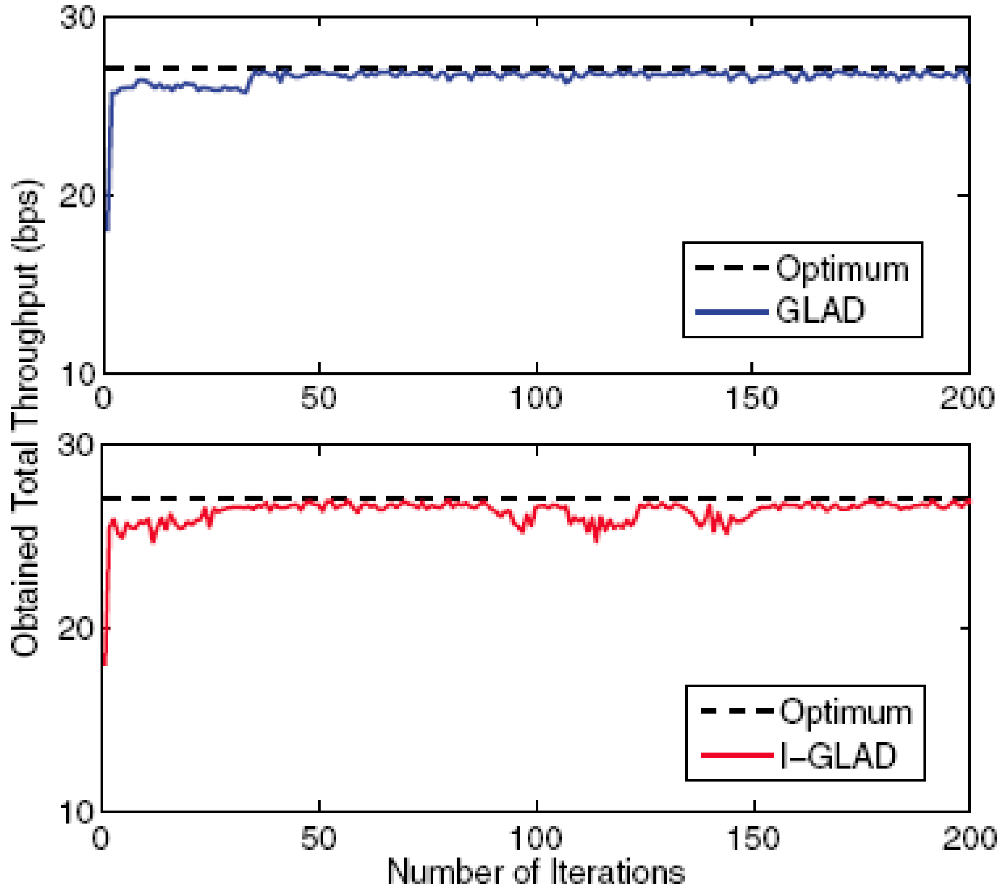




MRF Power Control Indicative Results (Throughput)

- Obtained total throughput with fading channels (10 links)
- Obtained total throughput w.r.t. # of iterations



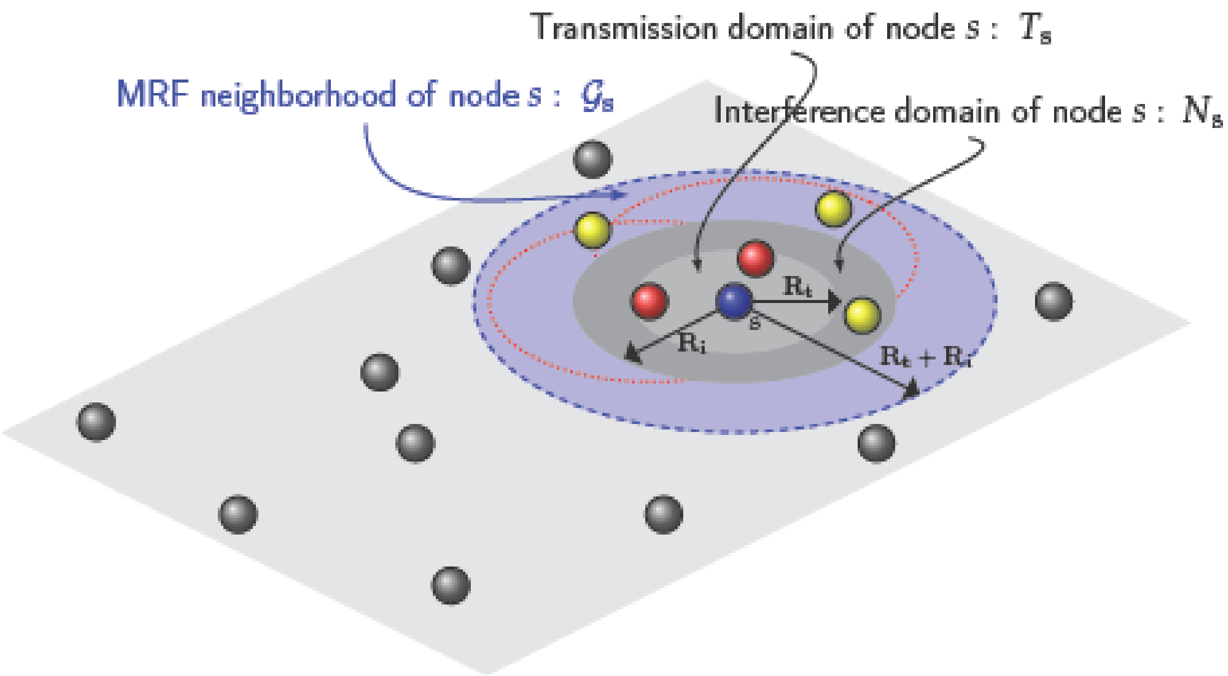


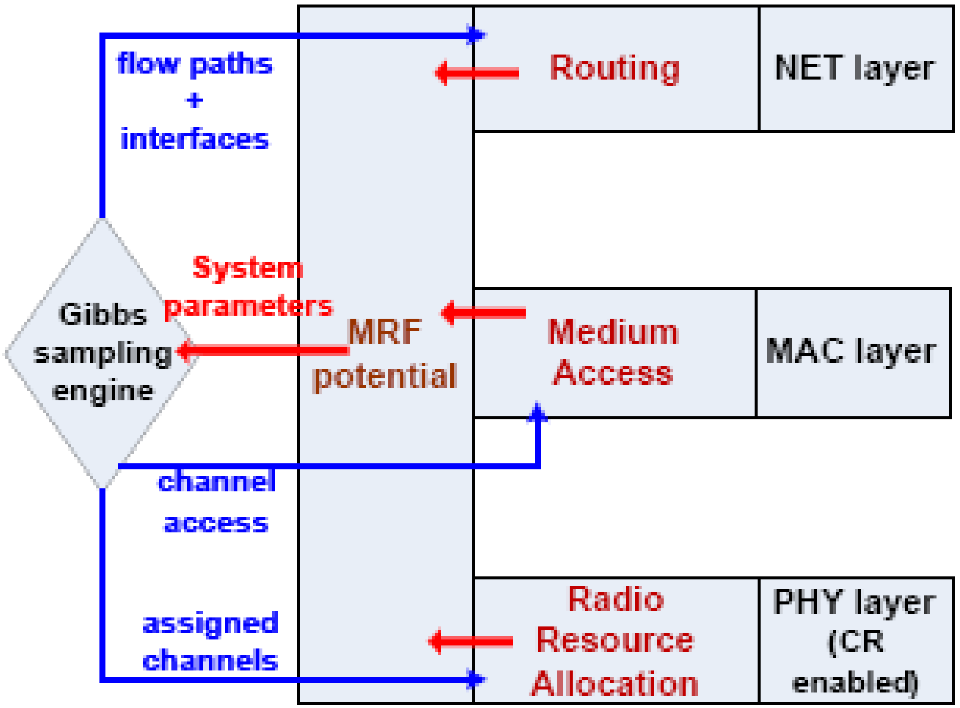
MRFs and Cross-layer Design in Complex Networks

- **General problem**: model resource allocation in complex networks
  - Various emerging tradeoffs with protocol layer inter-dependencies
  - Resources are scarce and highly desired by all users
  - Complexity should be as low as possible
  - Unifying framework for complex topologies
- **Critical challenges:**
  - Keep signaling low – local only information exchanges
  - Distributed computation/operation (or at least semi-distributed)
  - Ensure QoS guarantees
- **Specific problem:** PHY-MAC-NET resource assignment in CRNs
  - Previous approaches depend on heavy optimization
  - Mostly centralized with significant overhead

MRFs and Cross-layer Design in CRNs

- Cognitive radio network: Primary (centralized) – secondary users (distributed)
  - Determine resources of SU given activity of PU: channels – access – path augmentation
- **Main concept:** decompose decision from representation
  - Design them in a modular fashion – case of MRFs here





CRN MRF Cross-layer Design Formulation – PHY

- State of nodes: list of assigned channels $\left(u^{(1)}\ u^{(2)}\ \ldots\ u^{(M)}\right)^T$ $u^{(m)} \in \Theta = \{0,1\}$
- Neighborhood system in range R_i
- Potential function → pairwise nearest neighbors

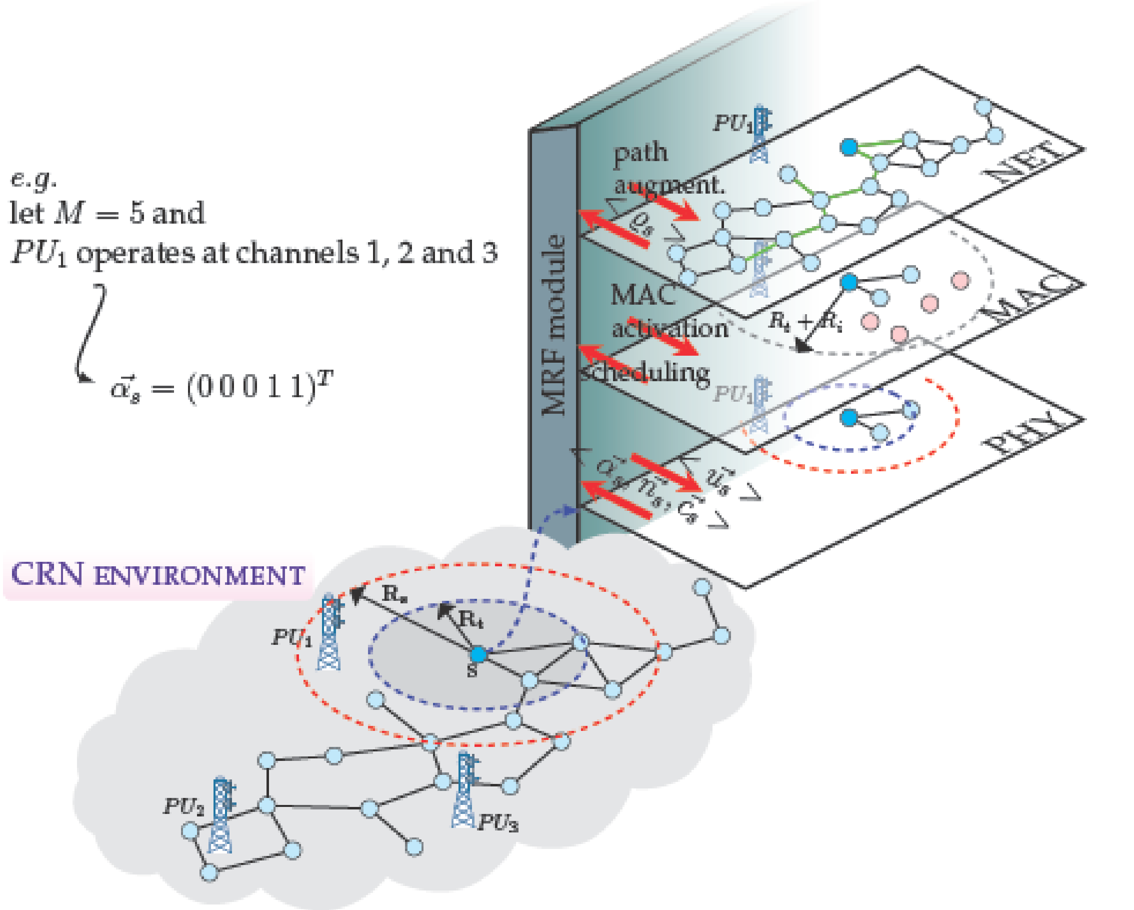
$$U(\omega) = \sum_{s \in S} V^{(1)}_{\{s\}}(x_s) + \sum_{\{s,j\} \in (S \times S), j \in \mathcal{G}_s} V^{(2)}_{\{s,j\}}(x_s, x_j)$$

$$V^{(1)}_{\{s\}}(x_s) = \begin{cases} \lambda_1 \cdot (1 - sig(\|\vec{u_s}\|_1)), & \text{if } \|\vec{u_s}\|_1 \geq 1 \text{ and } \\ & \vec{u_s} \cdot \vec{\alpha_s} = \vec{u_s} \cdot \vec{1} \\ \delta_1 > 0, & \text{otherwise.} \end{cases}$$

$$V^{(2)}_{\{s,j\}}(x_s, x_j) = \begin{cases} \lambda_2 \cdot \vec{u_s} \cdot \vec{u_j}, & \text{if } j \in T_s \text{ and } \\ & \vec{u_s} \cdot \vec{\alpha_j} \neq 0 \\ \frac{\lambda_2 \cdot \sum_{k \in T_s} (\vec{u_s} \cdot \vec{n_{sk}}) + \lambda_3 \cdot \vec{u_s} \cdot \vec{c_s}}{|\mathcal{G}_s \setminus \{T_s\}|}, & \text{if } j \in \mathcal{G}_s \setminus \{T_s\} \\ \delta_2 > 0, & \text{otherwise.} \end{cases}$$

e.g.
let $M = 5$ and
PU_1 operates at channels 1, 2 and 3

$\vec{\alpha_s} = (0\,0\,0\,1\,1)^T$



CRN MRF Cross-layer Design Formulation – MAC/NET

- MAC scheduling: determined by doubleton potential
  - If channels suffice → orthogonal assignment
  - If channels insufficient → CSMA/CA scheme

$$\breve{T}(w) = \frac{c_o}{\ln(1+w)}$$

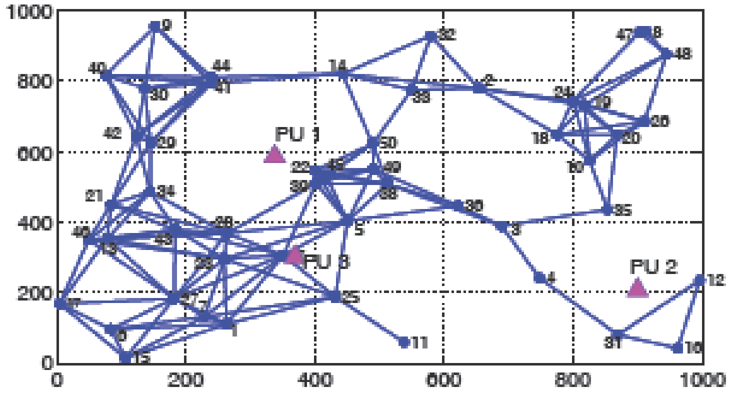
- NET path augmentation: enhance available paths with additional channels
  - Cross-layer metric *ρ(s)*
  - Routing protocol agnostic
  - Pushes information from NET-to-PHY
  - Higher-quality routing paths

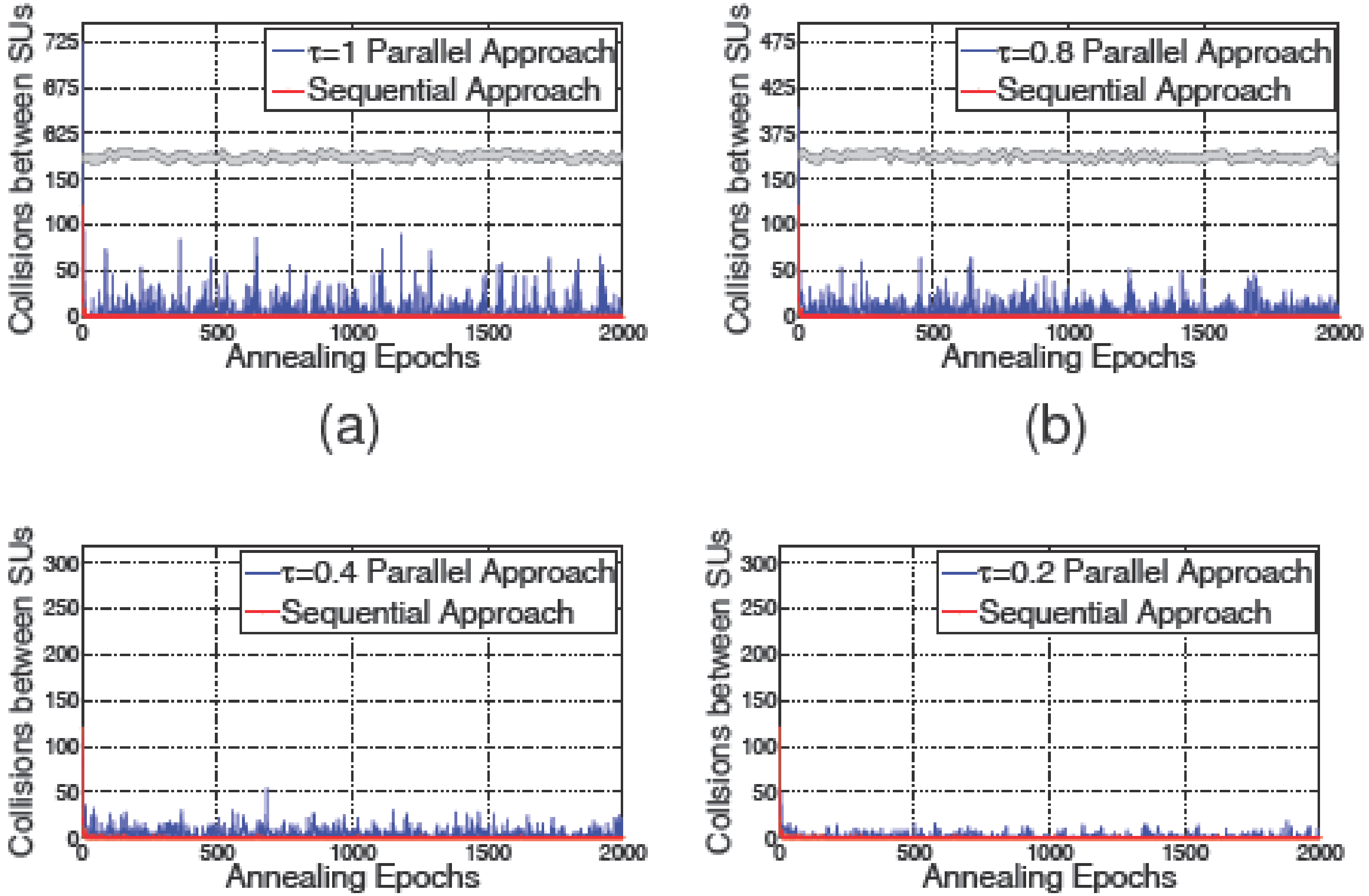
$$V_{\{s\}}^{(1)}(x_s) = \begin{cases} \lambda_1 \cdot \left(1 - \dfrac{A}{1 + 1 \cdot e^{-C\left(x - d - \varrho(s) \cdot step\right)}} - B\right), \\ \qquad \text{if } \|\vec{u_s}\|_1 \geq 1 \text{ and } \vec{u_s} \cdot \vec{\alpha_s} = \vec{u_s} \cdot \vec{1} \\ \delta_1 > 0, \quad \text{otherwise.} \end{cases}$$

- Semi-parallel implementation
  - Sites update state independently w.p. *τ*
  - Reduces mean sweep time *(nτ)*

MRF Cross-layer Indicative Results – Convergence

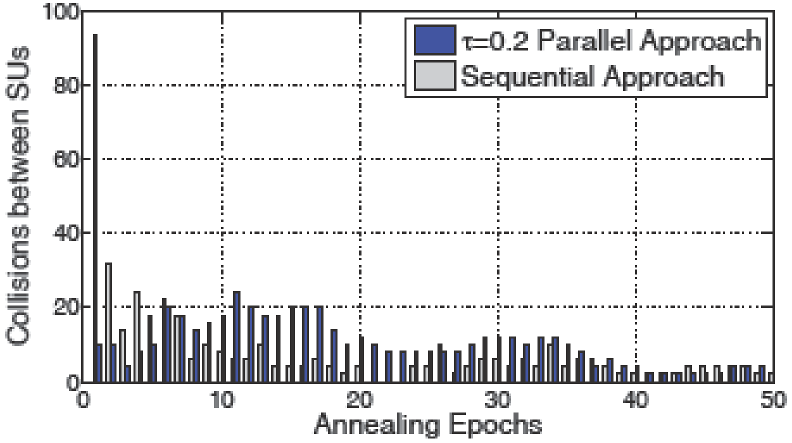
- Random topology
- Effect of τ on convergence



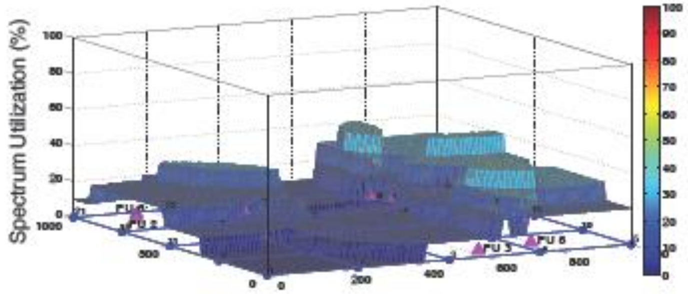


MRF Cross-layer Indicative Results – Resource Allocation I

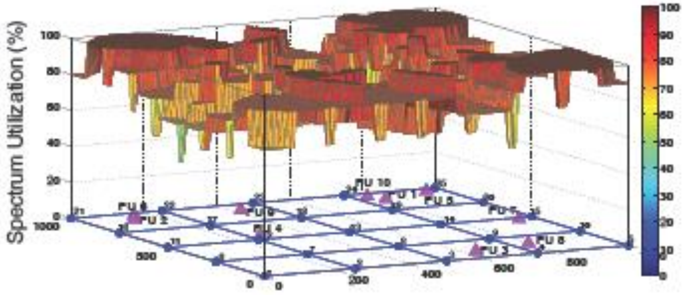
- Av. # of SU using CSMA/CA
- Spectrum utilization in space domain
- Collision avoidance among SUs



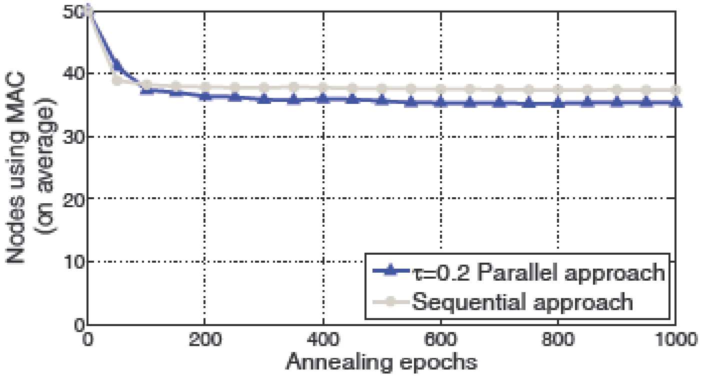
(a)

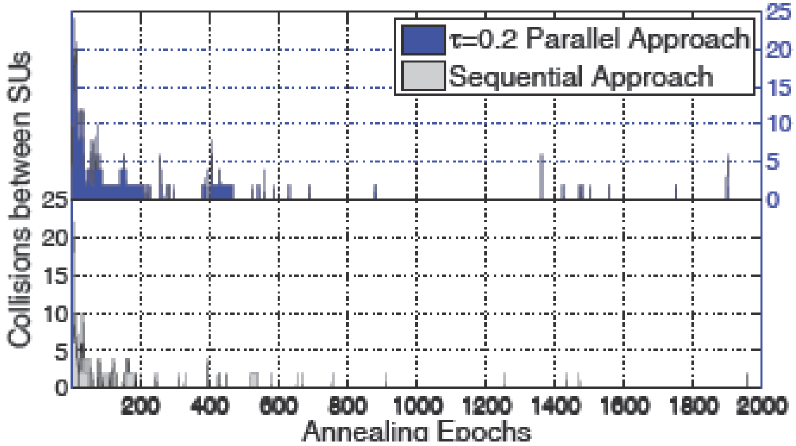


(a) Active primary network - inactive SUs.



(b) SUs exploit sequential MRF approach.

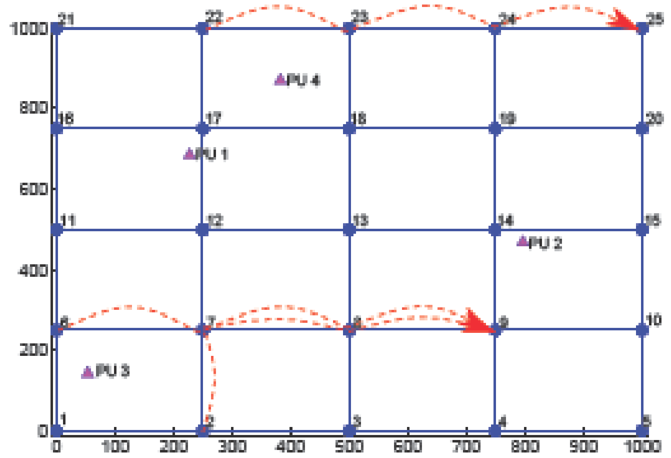


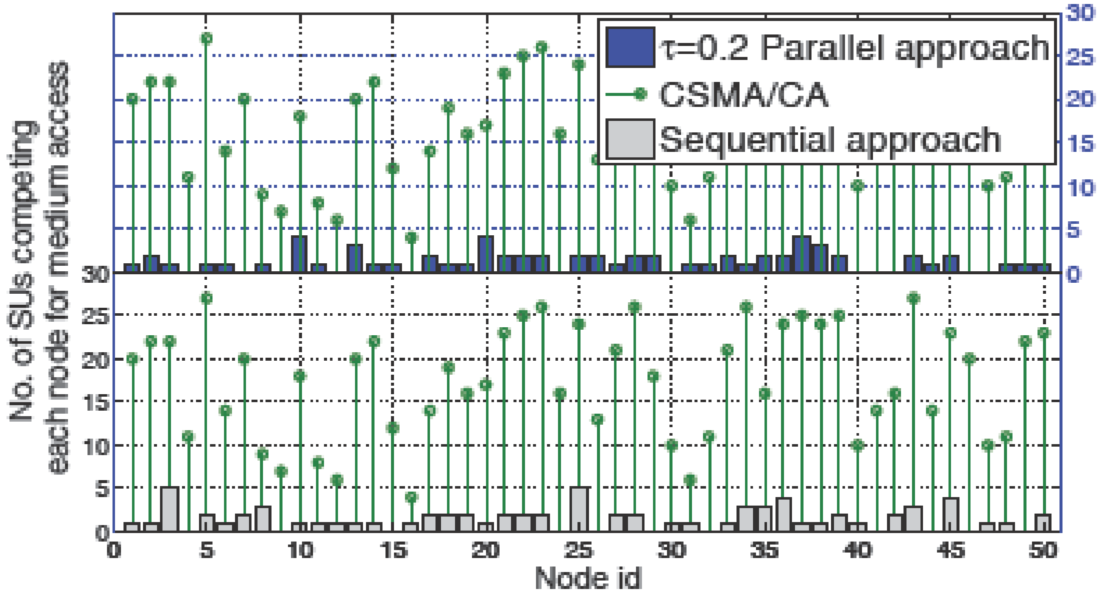


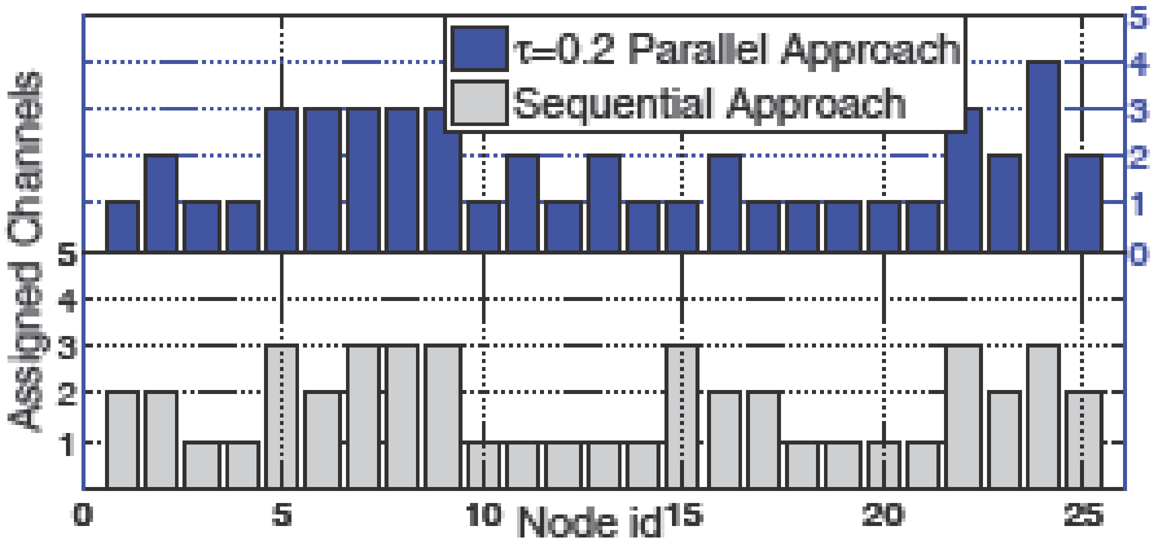
(b)

MRF Cross-layer Indicative Results – Resource Allocation II

- Comparison of pure CSMA/CA with sequential and semi-parallel approaches
- Grid scenario with 2 multiplexed flows
- Channel assignment to specific nodes
  - Nodes, 7, 8, 9 favored due to serving flows







MRFs in the Future: Challenges

- Dynamic MRFs
  - MRF models for scenarios with site churn: is it possible somehow?
  - Convergence?
- Networks with mobility
  - Currently impossible to achieve convergence
  - Slowly-varying mobility viable
- Sequential – semi-parallel – parallel implementations
  - Overhead vs. accuracy and convergence guarantees
- Identify objective-‘optimal’ annealing schemes

Potential Applications of MRFs

- Information diffusion and social network analysis
  - Combined with resource allocation in centralized types of networks, e.g. spectrum database CRNs
- Reputation and trust
  - Model and manage confidences and their shaping factors
- Voting and opinion formation: voting prediction systems
  - Simulate the strength/acceptance of tendencies in forthcoming public votes
- Distributed computation
  - Approximations of various forms of computation in distributed agents
- Fast decision-making under risk-prone environments
  - Applications in portfolio management, etc.

Summary

- Complex networks and Network Science
  - A multi-disciplinary unified theory for studying/designing/engineering networks
- Markov Random Fields (MRFs)
  - Statistical mechanics approach allowing stochastic optimization and much more…
- Various applications in communications networks
  - Swarming :: malware propagation :: power control :: cross-layer design
- Low operational/implementation complexity
- Very close to optimal solutions – often global optimal solutions
- Sequential and parallel implementations
  - Depending on applications and scenarios
- A lot of potentials for future applications

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Thank you for your attention!

??? Questions ???

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… comments …